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SHORTENING THE COURSE IN ARITHMETIC.*

By Sherman Williams.

The great amount of time given to the study of arithmetic, and the very meager results secured, is not after all due so much to the extent and variety of the work attempted as to inefficient teaching, though much of the work might be omitted without serious loss. All the work in arithmetic might be included under three heads.

1. *Memorizing.*—Under this head would come the learning of the tables of addition, multiplication, tables of denominate numbers, the aliquot parts of a dollar and the like. All this work, and a considerable other work in arithmetic could be done in the first four years of school without taking, on the average, more than ten or fifteen minutes a day.

2. *Acquiring Processes.*—This work should be completed by the end of the sixth year, and not more than twenty minutes a day need be given to the work during the fifth and sixth years of school life. The acquisition of processes should be chiefly memory work, reasons should not be required or given.

3. *A Study of the Principles Underlying the Processes.*—This is really high school work though something can be done in the seventh and eighth years. At the close of the sixth year the child should have a good working knowledge of arithmetic, sufficient to serve him in the common affairs of life. The later work, if undertaken, is disciplinary and does not help greatly in the application of arithmetic to the affairs of everyday life.

Suppose we consider separately the three phases of work that have been indicated and see if we can discover where mistakes are commonly made and where time can be saved. There are a few fundamental principles that must constantly be kept in mind.

1. Children like to do the things that they think they can do well.

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* Read at the meeting of the Syracuse Section.

2. Progress is easier and more rapid when the separate steps are short.

3. Only one step should be taken at a time.

4. Things that need to be permanently known must be fixed through a great deal of drill that extends over a considerable period of time.

5. Confidence is a great source of inspiration and power and should be judiciously cultivated.

6. Nothing that one thoroughly understands is difficult.

7. Children will accomplish little or nothing unless they are interested.

8. There can be no interest without attention.

9. No child can give attention for a long period of time.

For the first four years of school life no period of recitation should exceed fifteen minutes—ten minutes would be far better. Nowhere below the high school should the recitation period exceed twenty-five minutes, and the subject must be pretty interesting, and the teacher pretty skilful to hold the attention of any class for twenty-five minutes.

There is comparatively little difficulty in teaching children to write and read numbers. The first trouble is in teaching children to add, yet it is one of the easiest things to teach children to add so that they will never make a mistake through ignorance—they may through carelessness. No one ever makes a mistake in adding two and two. This is one combination. There are only forty-five, and one is no more difficult than another, and a child who learns one combination perfectly can learn them all perfectly, and yet mistakes in adding are very common. No one ever really adds. One sees or hears 3 and 5 and recalls 8. It is a case of purely arbitrary memory. Any attempt to develop the addition table through the use of objects is foredoomed to failure. It is not learned that way and cannot be no matter how skilful the teaching, or how earnest the efforts of the pupils. The use of objects in teaching addition is rarely if ever helpful, and very commonly harmful. It very often leads to the pernicious finger-counting habit, which is so difficult to overcome.

Children make mistakes in adding simply because of inefficient teaching. They are drilled upon the combinations which they already know perfectly just as much as upon those which they do not know. They are not trained to have confidence in themselves. To some extent they are trained to not have confidence. Often they are told to add up a column of figures, and then to be sure that they are right add the column downwards. Often the two additions will not agree and confidence is gone. Children should not be told when they have made a mistake in adding. The teacher should observe what combination it is that the child does not know and then give much drill upon it.

There should be some system in the work in addition. It is not a bad plan to take first those combinations the sum of which does not exceed nine. These constitute twenty of the forty-five combinations. It will be found that there will be most difficulty with the 6s, 7s, 8s and 9s. There should be most drill upon these.

There are many devices for drill in addition. Counting to 100 by 1s, 2s, 3s, 4s, 5s, 6s 7s, 8s and 9s. Counting both forward and backward and beginning with 1, 2, 3, 4, 5, 6, 7, 8 or 9, and in counting backward beginning with 100, 99, 98, 97, 96, 95, 94, 93, 92 or 91. Counting by series, both direct and broken. Sometimes a serious mistake is made by asking the child when adding a column of figures to notice that 4 and 6 are ten, 7 and 3 are ten, etc., and to take the tens separately. This is giving drill upon the combinations well known, and therefore easy, rather than upon the combinations less well known—that is putting the drill where it is least needed.

Never allow a child to guess at an answer. He either knows it or he does not. It is a case of arbitrary memory. It is not a matter that can be thought out. He is as well off if he guesses wrong as though he guessed right, possibly better off.

A very effective method of drill is to have the whole class add together silently, first giving the numbers so slowly that the dullest child in the class can do the work, then gradually increasing the speed till the brightest and quickest child is taxed to his utmost. This is work that children like. Every child can do a part of it. Each one tries to keep up with the class a little longer each day.

It is a very easy matter to drill the children so that they all will have learned all the combinations in addition by the end of the third year. They will have learned much else beside.

It is not well for a child to be constantly meeting things wholly new. This should be carefully guarded against. The subtraction table should be made more or less familiar through the study of the addition table, and in like manner much of division should be learned while studying multiplication, and the study of division should lead to an understanding of fractions. When a child learns that 5 and 4 make 9 he should be required to present that fact in a different way as 5 and what make 9. 4 and what make 9. There should be much of this kind of work in connection with addition, and similar work leading to division in connection with multiplication. The multplication table will be learned in much the same way as the addition table and quite as easily.

Before any written work is attempted there should be much mental work involving addition, subtraction, multiplication, and division. Most processes are most easily learned through mental work. Of course in a strict sense all arithmetical work is mental.

The learning of processes should be mainly a matter of memory. No explanations should be given at the time the processes are learned. That is a work that demands a greater maturity of mind than most children possess at this time. In all the early work in arithmetic a good model should be given and it should be followed perfectly, no variation being allowed. Processes should be fixed in mind by much repetition.

The first written work in addition should consist of examples in which the sum of no column should exceed nine. Keep up practice with such examples till the pupils are both accurate and rapid in their work, then introduce the matter of carrying but do not attempt to explain it. It is quite enough to say that people add that way. That is all the explanation that there is for a great deal of the work that comes up in school. People do so and so. They might do differently but they do not.

If the oral work in addition is well done there will be little trouble with the written work. The same is true of all the fundamental operations.

I see no choice in the two methods in subtraction in the case in which the number of the subtrahend is larger than the corresponding number of the minuend, but whichever method is chosen should be adhered to up to the seventh grade.

Only one step should be taken at a time, and that the shortest possible one. This principle is violated oftener than almost any other. Take multiplication for example. It is no uncommon thing for a teacher to begin the written work by giving an example in which the multiplier has two figures and the multiplicand three or four. Here are three steps at once instead of one, multiplying a single number, carrying and dealing with partial products. Is it to be wondered at that with work done in this manner many of the pupils become hopelessly confused?

The same kind of a mistake is made in teaching division, and with even more disastrous results.

When the four fundamental rules have been mastered there is little else to be learned in arithmetic. There is merely a difference in form and terminology, or nomenclature. There are no new principles.

After the completion of the fundamental rules and before beginning the work in fractions the pupils should be led to see clearly that multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient. It is not enough that they be shown this and that they are able to state the fact glibly, but they must actually see it as the result of many problems worked by them, the work being carried over a considerable interval of time. The clear understanding of the work in fractions depends upon a perfect understanding of this principle. Two great difficulties in the teaching of arithmetic are, first, that the pupils pass on to more advanced work before they really fully understand the fundamental operations. Second, that teachers make difficulties for them by telling them they are to take up something new when in point of fact no new principles are involved, and in addition to this they make several cases as in fractions, percentage, etc., when the whole work should be regarded as a unit with pupils below the seventh grade.

A fraction is simply an indicated division expressed in a way new to the children, and the children should be told this at the outset. So decimal fractions are merely another form of expressing indicated division. It is a pretty good plan to make the children quite familiar with decimals while teaching the fundamental rules, without saying anything about their being decimals, or differing in any way from the numbers they have been dealing with save in the matter of value. There really is no other difference.

Children are often taught that they are entering upon a new and different field when they begin the study of fractions, but of course this is not true, and they ought not be allowed to think there is anything new save form and nomenclature. They have been accustomed to add concrete things of the same kind and know that they cannot add unlike things. The same should be seen to be true in fractions. They have added apples to apples, and pears to pears, but not apples to pears. So they may add fourths to fourths and fifths to fifths, but not fourths to fifths.

What is to be done with such numbers. A fraction is always an indicated division. What they called the dividend in whole numbers they call the numerator in fractions, and what was known as the divisor in whole numbers is called the denominator in fractions. The children know that in any individual case they may multiply both divisor and dividend by the same number without changing the value of the expression. So with numerator and denominator. The fifths may be made twentieths by multiplying both numerator and denominator by the number four. The fourths may be made twentieths by multiplying both numerator and denominator by five, and twentieths can be added. With this matter clear there is no further trouble with fractions, or at any rate any fractions that may properly be considered before the seventh year, if you proceed slowly, taking only one step at a time. There is no need of any plan of finding the least common multiple. That can be seen by inspection. It is always the largest denominator or some number of times that number. There should be a great amount of mental work in fractions. All kinds of work in fractions should be made clear through simple mental work.

During the fifth and sixth years there should be a great number of practical examples dealing with real conditions, and covering a wide range of work. There should be many simple examples in mensuration, and each example may be made to cover more than one point after the single steps are clear, for example a bin is so long, wide, and deep. How many cubic feet does it contain? How many bushels will it hold, how many barrels, how many pounds of wheat? Have pupils measure things about them, piles of wood, floor space, walls, etc. Then have them work with measurements given them, but in all cases have them substitute drawings or diagrams for the real thing, continuing that practice till they have formed the habit of visualizing.

The whole range of work in percentage can be made perfectly clear through oral work. Have much drill in such examples as these:

6 is what per cent. of 18?

What per cent. of 20 it 5?

1/3 of 18 is what per cent of it?

5 per cent. of 20 is what per cent of 10?

12 per cent. of 200 is 10 per cent. of what number?

In teaching percentage, as in teaching fractions, difficulties are created by making many different cases where there is really only one. Of course there are minor differences that you may recognize in dealing with older children, but with the younger ones it merely serves to confuse.

I have outlined in a general way the work that can easily be completed by the end of the sixth year, and done well and thoroughly without the expenditure of any very great amount of time. It will give the pupils all the knowledge of arithmetic that ninety-nine out of a hundred will ever have any occasion to use. In my judgment the study of arithmetic should not be carried beyond what I have indicated until the latter part of the high school course. If one wishes something may be done in the seventh and eighth grades, but it seems to me that the time can be used to a much better advantage with other subjects.

So far I have said nothing about omissions save by implication. I am loath to attempt it. There is sure to be no general agreement upon this point. An argument can be made for the retention of any given topic, or for the introduction of almost any. They can be shown to be of some value. But the real question is not, is this or that of value, but is it at this point in the school course of greater value than some other subjects.

In teaching any subject, in any grade, the question is not is it of value, and can the work be done, but can it be done at a greater profit to the pupils than some other work.

With these thoughts in mind I will venture to express the belief that in the first six years of school life all examples should consist of small numbers and be expressed in the simplest way, and that the greater part of the examples should be those that can be solved mentally. I believe that little or nothing should be taught in regard to the least common multiple or the greatest common divisor. I believe that absolutely no work should be required in proportion, compound partnership, partial payments, true discount, average of accounts, arithmetical progression, geometrical progression, square or cube root.

With the elimination of these subjects and simple treatment of the others there is no reason why wholly satisfactory work may not be done with a small expenditure of time, and the work completed at the end of the sixth year. This can be done in any school where there is a competent teacher, as well in the most remote rural school as in the best village or city school.

Glens Falls, N. Y.

"If you and I—just you and I—
Should laugh instead of worry;
If we should grow—just you and I—
Kinder and sweeter hearted,
Perhaps in some near by-and-by
A good time might get started;
Then what a happy world 'twould be
For you and me—for you and me."